

# MRF Solutions for Probabilistic Optical Flow Formulations

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## Problem : Optical Flow Estimation

- Dense flow field
- Small motions (one pixel or less)
- Non-parametric motion model (arbitrary motions)
- Contant Brightness Assumption (CBA)

## Contribution: New Optical Flow Algorithm

- Probabilistic interpretation of the CBA
- Novel parametrisation of optical flow
- Efficient and Global solution via graph-theoretic formulation

## Model

Spatio-temporal derivatives are noisy.

A probabilistic interpretation is needed!

Spatio-temporal derivatives :  $I_d$  ( $I_d = [I_x, I_y, I_t]$ )

True spatio-temporal derivatives :  $I_d^0$

Noise Model :  $I_d = I_d^0 + n$   $n \sim N(0, \Sigma)$

Motion :  $v = [v_x, v_y, 1]$

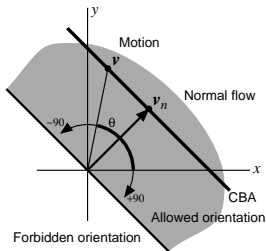
Motion, given the image derivatives :  $P(v|I_d)$

$$P(v|I_d) = P(v|I_d^0)P(I_d^0|I_d)$$

Conditional prior on the motion    Noise model :  $N(I_d, \Sigma)$

## Selecting a conditional motion prior

"All flow orientations consistent with normal flow are equally likely"



$$P(\theta|I_d^0) = \frac{1}{\pi}, |\theta - \theta_n| \leq \frac{\pi}{2}$$

$$0 \text{ otherwise}$$

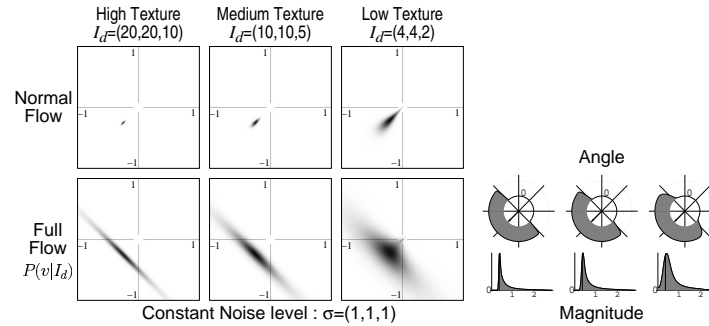
Since

$$v = [\cos(\theta), \sin(\theta)] \|v_n\| \sec(\theta - \theta_n)$$

this directly defines  $P(v|I_d^0)$

In practice, the distributions are obtained by sampling the derivatives.

## Typical motion distributions from our model



## Are Orientation and Magnitude independent?

Experimentally supported by the correlation coefficient.

$$P(v|I_d) = P(\theta|I_d)P(m|I_d) \quad \rho_{\theta m} = 0.04$$

$$P(v|I_d) = P(x|I_d)P(y|I_d) \quad \rho_{xy} = 0.4$$

Almost independent!

## Resolving the optical flow : An MRF Formulation

- Motion is under-constrained by the CBA
- Added constraint : Neighbor pixels have similar motions
- Ideal for a Markov Random Field formulation
- The single parameter + linear penalty MRF can be solved globally and efficiently via a maximum-flow computation in a graph

### 1. Solve for orientation only

$$\min_{\theta} \sum_{i \in S} \sum_{i' \in N_i} \beta |\theta_i - \theta_{i'}| - \sum_{i \in S} \ln(P(\theta|I_{d_i}))$$

Orientation is discretized in steps of 1 to 4 degrees

### 2. Solve for magnitude, given the orientation

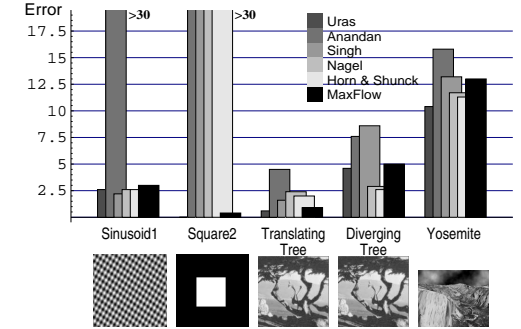
Use  $P(m|\theta_{si}, I_{d_i})$  rather than  $P(m|I_{d_i})$

$$\min_{m} \sum_{i \in S} \sum_{i' \in N_i} \beta |m_i - m_{i'}| - \sum_{i \in S} \ln(P(m|\theta_{si}, I_{d_i}))$$

Magnitude is discretized from 0 to a maximum value (~2 pixels) by steps of 0.1 pixel

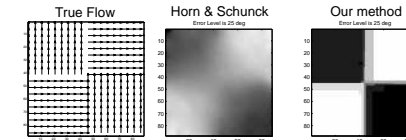
## Results : Synthetic Images

Comparison with dense results from [Barron et al., 1994]



## Results : Recovering Flow Discontinuities

- Create a discontinuous flow pattern ([Barron et al., 1994] has only smooth fields)
- Solve with two different algorithms up to a common level of orientation error : 25°
- Our algorithm does not smooth over discontinuities
- Our algorithm enforce smoothness by introducing a bias



## Results : Real Images

